DOCUMENT RESUME

ED 070 660 24 SE 015 469

AUTHOR Harris, Margaret L; Romberg, Thomas A.
TITLE An Analysis of Content and Task Dimensions of Mathematics Items Designed to Measure Level of Concept Attainment.
INSTITUTION Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.
SPONS AGENCY National Center for Educational Research and Development (DHEW/OE), Washington, D.C.
REPORT NO WURDCCL-TR-196
BUREAU NO BR-5-0216
PUB DATE Nov 71
CONTRACT OEC-5-10-154
NOTE 42p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS *Cognitive Ability; Content Analysis; *Elementary School Mathematics; *Evaluation; Factor Analysis; *Mathematical Concepts; Mathematics Education; Task Analysis; *Test Construction

ABSTRACT

Mathematics items developed to measure concept attainment for the topics of set theory, division, and expressing relationships were studied. A completely crossed design with 30 concepts and 12 tasks was used. The items were administered to 196 girls who had just completed the fifth grade and to 195 boys who had just begun the sixth grade. Conventional factor analyses were performed, separately for boys and girls, for concept scores and for task scores; these showed that all 30 of the concepts were measures of a single functional relationship existing among the concepts and that all 12 tasks were measures of a single underlying ability or latent trait. Three-mode factor analyses indicated that there were no important concept-task interactions for the idealized person and thus the concepts and the tasks can be regarded as being two independent modes. For related documents in this series, see SE 015 462 and SE 015 468. (Author/DT)

ED 070660

PR 5021
SE

TECHNICAL REPORT NO. 196

AN ANALYSIS OF CONTENT AND TASK DIMENSIONS OF MATHEMATICS ITEMS DESIGNED TO MEASURE LEVEL OF CONCEPT ATTAINMENT

REPORT FROM THE PROJECT ON A STRUCTURE OF CONCEPT ATTAINMENT ABILITIES

U.S. DEPARTMENT OF HEALTH
EDUCATION & WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

FILMED FROM BEST AVAILABLE COPY

WISCONSIN RESEARCH AND DEVELOPMENT

CENTER FOR COGNITIVE LEARNING

THE UNIVERSITY OF WISCONSIN
Madison, Wisconsin

U. S. Office of Education
Center No. C-03
Contract OE 5-10-154

RESEARCH
COGNITIVE LEARNING
DEVELOPMENT

SE 015 469

ED 070660

Technical Report No. 196

AN ANALYSIS OF CONTENT AND TASK DIMENSIONS OF MATHEMATICS ITEMS DESIGNED TO MEASURE LEVEL OF CONCEPT ATTAINMENT

By
Margaret L. Harris and Thomas A. Romberg

Report from the Project on
A Structure of Concept Attainment Abilities
Robert E. Davidson, Lester S. Golub, Herbert J. Klausmeier,
Thomas A. Romberg, B. Robert Tabachnick, Alan M. Voelker
Principal Investigators
and
The Quality Verification Program
Mary R. Quilling, Director

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

November 1971

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the United States Office of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the Office of Education and no official endorsement by the Office of Education should be inferred.

Center No. C-03 / Contract OE 5-10-154

Statement of Focus

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from the Project on the Structure of Concept Attainment Abilities in Program 1. The general objectives of this project are to identify basic concepts in language arts, mathematics, science, and social studies appropriate at a given grade level; to develop tests to measure achievement of these concepts; and to develop and identify reference tests for cognitive abilities. These will be used to study the relationships among learned concepts in various subject matter areas, cognitive abilities, and possibly, certain cognitive styles. The results of these will be a formulation of a model of structure of abilities in concept attainment.

Contents

		Page
	List of Tables and Figures	vii
	Abstract	ix
I	Introduction	1
	Nature of Mathematics Items	1
	Hypothesized Factor Structures	3
	Concepts	3
	Tasks	3
II	Procedures	5
	Subjects	5
	Data Collection	5
	Treatment of the Data	6
	Factor Analysis	7
III	Results and Discussion	13
	Reliability Estimates and Test Statistics	13
	Factor Analyses	14
	Conventional Factor Analyses	18
	Three-Mode Factor Analyses	25
IV	Summary and Conclusions	29
	References	31
	Appendices	33

List of Tables

Table		Page
1	Mathematics Concepts Categorized by Area	3
2	Test Data for Population and Samples	6
3	Distribution of Fathers' Occupations	7
4	Means, Standard Deviations, and Reliabilities for Mathematics Concept and Task Scores: Boys and Girls	14
5	Reliability Estimates for Task Scores by Area and Total for Girls	15
6	Intercorrelations of Mathematics Concepts: Boys	16
7	Intercorrelations of Mathematics Concepts: Girls	17
8	Intercorrelations of Mathematics Tasks: Boys	18
9	Intercorrelations of Mathematics Tasks: Girls	18
10	Numbers of Initial and Derived Factors for Concept Scores: Boys and Girls	19
11	Numbers of Initial and Derived Factors for Task Scores: Boys and Girls	19
12	Oblique Common Factor Results for Mathematics Concepts: Boys	20
13	Oblique Common Factor Results for Mathematics Concepts: Girls	21
14	Oblique Common Factor Results for Mathematics Tasks: Boys	22
15	Oblique Common Factor Results for Mathematics Tasks: Girls	22
16	Three-Mode Core Results: Boys	27
17	Three-Mode Core Results: Girls	28

List of Figures

Figure		Page
1	Item Matrix for Each Individual	8

Abstract

Content and task dimensions of mathematics items were studied using factor analytic techniques. These items were developed to measure concept attainment using a completely crossed design with 30 concepts and 12 tasks. Conventional factor analyses were performed, separately for boys and girls, for concept scores and for task scores. Three-mode factor analyses were performed.

The main conclusions drawn from the results of the conventional factor analyses are that all 30 of the concepts are measures of a single functional relationship existing among the concepts, and that all 12 tasks are measures of a single underlying ability or latent trait. The three-mode results indicate that there are no important concept-task interactions for the idealized persons; thus it is reasonable to regard the concepts and the tasks as being two independent modes.

I
Introduction

The primary objective of the project entitled "A Structure of Concept Attainment Abilities" (hereafter referred to as the CAA Project) is to formulate one or more models or structures of concept attainment abilities, and to assess their consistency with actual data. The major steps for attaining this primary objective were taken to be:

1. To identify basic concepts in language arts, mathematics, science, and social studies appropriate at the fourth grade level,
2. To develop tests to measure achievement of these concepts,
3. To identify reference tests for cognitive abilities, and
4. To study the relationships among learned concepts in these four subject matter fields and the identified cognitive abilities.

This paper contains a report of the factor analytic study of the content and task dimensions of the mathematics items that were developed as one aspect of Step 2. This study is a necessary intermediate step between Step 2 and Step 4; some reduction in the number of concepts for each subject matter field from the 30 selected ones for which tests were developed is mandatory in order to be able to carry out Step 4.

Nature of Mathematics Items

Concepts may be defined in one or more of four ways: (a) structurally, in terms of perceptible or readily specifiable properties or attributes; (b) semantically, in terms of synonyms or antonyms; (c) operationally, in terms of the procedures employed to distinguish the concept from other concepts; or (d) axiomatically, in terms of logical or numerical relationships (Klausmeier, Harris, Davis, Schwenn, & Frayer, 1968). "A concept exists whenever two or more distinguishable objects or events have been grouped or classified together and set apart from objects on the basis of some common feature or property of each" (Bourne, 1966, p. 1). The concept of Bourne's definition might be called a classificatory one and seems to be the same as the structural type discussed by Klausmeier et al. (1968). This is the type of concept with which this project is concerned, and such a definition of a concept served as the basis for selection and analysis of subject matter concepts.

Many different types of performance might be taken as the critical evidence that a student does or does not understand a given concept. Thus, as a part of this project it is necessary to have a schema for measuring understanding of concepts. Such a schema was developed by Frayer, Fredrick, and Klausmeier (1969) and was used by the CAA Project to assess concept attainment. The "Schema for Testing the Level of Concept Mastery" consists of 13 types of questions, each involving a different task required of the examinee. The schema also allows for selection of an answer (multiple-choice type questions) or for production of an answer (completion type questions). It was decided to use the first 12 tasks and a multiple-choice format for this project.. The 12 tasks of the schema which were used are:

1. Given the name of an attribute, select an example of the attribute.
2. Given an example of an attribute, select the name of the attribute.
3. Given the name of a concept, select

an example of the concept.

4. Given the name of a concept, select a nonexample of the concept.

5. Given an example of a concept, select the name of the concept.

6. Given the name of a concept, select the relevant attribute.

7. Given the name of a concept, select the irrelevant attribute.

8. Given the definition of a concept, select the name of the concept.

9. Given the name of a concept, select the definition of the concept.

10. Given the name of a concept, select the supraordinate concept.

11. Given the name of a concept, select the subordinate concept.

12. Given the names of two concepts, select the relationship between them.

Single- or compound-word classificatory concepts (those that are defined by attributes) in mathematics subject matter at the fourth grade level were identified. This task was subdivided into four steps:

1. Identification of the major areas within the subject matter of mathematics,

2. Selection of three of these major areas to be studied,

3. Identification of classificatory concepts within each of these three major areas, and

4. Random sampling of ten concepts from those identified for each of the three major selected areas.

This yielded a total of 30 mathematics concepts to be studied by the project. A list is given in Table 1, by area, of the concepts identified and randomly selected for study. The areas are Sets, Division, and Expressing Relationships. A description of the procedures used to identify these concepts can be found in "Selection and Analysis of Mathematics Concepts for Inclusion in Tests of Concept Attainment" (Romberg, Steitz, & Frayer, 1971).

The researchers of Project 101, Situational Variables and Efficiency of Concept Learning, developed a system for analyzing a concept in preparation for developing items to measure the level of attainment of that concept (Frayer, Fredrick, & Klausmeier, 1969). Since the publication of that paper they, in cooperation with the researchers of the CAA Project, have refined their thinking and advanced this system. The refinements are discussed in "A Structure of Concept Attainment Abilities: The Problem and Strategies for Attacking It" (Harris, Harris, Frayer, & Quilling, in press). Briefly, a concept may be described in many ways: in terms of its criterial, relevant, and irrelevant attributes; its examples and nonexamples; its supraordinate, coordinate, and subordinate hierarchical relationships (theoretically determined); and its lawful or other types of relationships to other concepts. Knowledge of each of these kinds of information may be tested to determine a student's level of attainment of a concept. An analysis, along these lines, of each of the 30 sampled mathematics concepts which are being studied can be found in "Selection and Analysis of Mathematics Concepts for Inclusion in Tests of Concept Attainment" (Romberg, Steitz, & Frayer, 1971).

Thus, using the analysis of a concept as the basis for appropriate content and the 12 tasks of the schema as the basis for appropriate tasks, 12 items, one for each of the 12 tasks, could be developed for each of the 30 concepts making a total of 360 mathematics items. Actually, only 353 items were developed for the purpose of measuring and assessing concept attainment in mathematics, as no subordinate concepts were identified for 7 of the 30 concepts studied; thus, there is no Task 11 item for those seven concepts. A description of the procedures used in the development of these items, along with item and total score statistics (for concepts and for tasks) obtained for them for fifth grade boys and girls, can be found in "Measuring Mathematics Concept Attainment: Boys and Girls" (Harris & Romberg, in press). The items can be found in "Items to Test Level of Attainment of Mathematics Concepts by Intermediate-Grade Children" (Romberg & Steitz, 1971).

The following sections contain a discussion of the study of the dimensionality of the two modes, concepts (content) and tasks, of this completely crossed design used to develop items to measure concept attainment in mathematics.

Table 1
Mathematics Concepts Categorized by Area

Set Theory	Division	Expressing Relationships
Cardinal Number	Algorithm	Area
*Disjoint Sets	Associative Property	*Average
Element	Closure Property	Dozenal System
*Empty Set	Common Denominator	Estimation
*Equal Sets	Commutative Property	Generating Sentences
*Equivalent Sets	*Denominator	*Graph
Intersection	Density Property	Length
*Line	Distributive Property	Liquid
Line Segment	*Division	Mathematical Sentences
Non-Disjoint Sets	*Factor	*Measurement
Ordered Pairs	*Fraction	*Open Sentence
*Parallel Lines	Identity Property	Partial Sums
*Plane	*Mixed Fraction	*Place Holder
*Point	*Multiplication	*Place Value
Set	*Numerator	Range
Sets of Numbers	Order Property	Round Numbers
Sets of Points	Partial Product	*Solution Set
Skew	Partial Quotient	*Standard Unit
*Subset	Partitioning	*Statement
*Subtraction - A way of looking at addition	*Product	*Weight
Triangular Numbers	*Quotient	
Union of Sets	Reciprocal Property	
Universal Set	*Remainder	
Whole Number		

*Concepts randomly selected to be tested.

Hypothesized Factor Structures

Alternative sets of factors were postulated for the mathematics concepts and for the tasks using mathematics content by viewing the concepts and tasks as two independent modes. Viewing them in this way is essentially hypothesizing that no important interactions exist between the two modes.

Concepts

The most general hypothesis is that just one common factor underlies the selected mathematics concepts. Next in the order of generality to specificity is that three common factors are present, one for each of the three major areas selected for study: Sets, Division, and Expressing Relationships. A more specific hypothesis is that there may be two or more common factors for each of the three areas. A structure of the concepts within each of the three areas was not hypothesized. Instead, it was preferred to randomly sample concepts from each area and see what functional relationships exist among those sampled concepts. It was felt that this would eliminate bias in the picture of the dimensionality of the concepts imposed by theoretical relationships that may or may not exist in actuality. If attainment of concepts is highly specific, this approach may be detrimental as there may not be at least two measures (concepts) of a concept dimension included. There are some indications that the concepts are not this specific. For example, fairly reliable task scores obtained by totalling across the 30 concepts for a single task were obtained. This indicates some degree of homogeneity among the concepts.

Tasks

The most general hypothesis is that just one common factor or ability underlies the 12 tasks. A more specific hypothesis is that

there are five underlying abilities: an ability dealing with attributes (Tasks 1 and 2), one dealing with examples of a concept (Tasks 3, 4, and 5), one related to the definition of a concept (Tasks 6, 7, 8, and 9), one related to hierarchical relationships (Tasks 10 and 11), and one for a relationship of a concept with another concept (Task 12). A slightly more specific hypothesis is that there are six abilities: the five just listed, with the exception that the ability related to the definition of a concept may be further specific to those tasks dealing with relevant and irrelevant attributes (Tasks 6 and 7) and those tasks dealing directly with a definition (Tasks 8 and 9).

These alternative sets of factors represent an a priori analysis of the mathematics concepts and the tasks when using mathematics content. A major question to be answered in this study is the extent to which the obtained factors parallel such hypothesized analyses. Note that, as discussed, several levels of specificity are postulated. Another question to be answered in this study is the extent to which the concepts and the tasks are independent as hypothesized.

II
Procedures

Subjects

Pilot studies revealed that the concepts selected were very difficult for fourth graders. Thus, the decision was made to test fifth grade students with the concepts identified and sampled from the fourth grade textbooks. The mathematics items were administered during early summer, 1970, to 196 girls who had just completed the fifth grade and to 195 boys who were just beginning the sixth grade during the fall of 1970 in the public school system of Madison, Wisconsin. The students were randomly selected from the population of all such girls and from the population of all such boys. The Madison Public School System made available the information concerning the population and used their computing facilities to designate the random sample for the girls.

Initially, a random sample of 300 girls was drawn. Letters were sent to the parents of these students explaining the purpose and details of the testing, and inviting their daughter to participate in the testing program. A stamped and addressed postcard was enclosed which the parents were asked to complete and return indicating whether or not they were willing to allow their daughter to participate. One hundred and two yes responses and 25 no responses were obtained from the cards returned. Those parents who had not returned the card by a specified date were phoned. An additional 46 yes and 61 no responses were obtained by phone. Since this total of yes responses did not give as many subjects as were desired, an additional sample of 150 girls was drawn at random. From this sample, 56 yes and 30 no responses were obtained by card. Thus, of the total sample of 450 students, 203 yes and 116 no responses were received; seven students did not complete the testing, which resulted in a total of 196 girls tested. These students were paid $7.50 for participating.

A random sample of 756 boys was drawn and letters were sent. By mail, 420 yes and 87 no responses were obtained. Thirty-eight of the subjects did not complete the testing, resulting in 382 boys tested. Of this total, 195 boys completed the mathematics and social studies items; the others responded to language arts and science items. As with the girls, the boys who completed the testing program were paid $7.50.

Since the participation of all students comprising the random sample was impossible to attain, test score and IQ data were obtained from the files of the Madison school system for both the school population and those participating students for whom the information was available. Table 2 includes the summary statistics for the population of fifth grade students in the public school system of the city of Madison during the school year 1969-1970, and for the boys and the girls who comprised the tested samples for the mathematics items. The IQs were obtained in the fall of 1968 when the subjects were fourth graders using the Lorge-Thorndike Intelligence Test, and the scores on the Iowa Tests of Basic Skills, given in grade equivalent scores, were obtained in the fall of 1969 when the subjects were fifth graders.

Data on fathers' occupations were collected from the students using the Master Occupational Code of the United States Bureau of the Census. These data were tabulated and are presented in Table 3.

Data Collection

The data for the girls were collected in two centrally located schools, one on the East side and one on the West side of the city, during five 2-hour daily sessions for

Table 2
Test Data for Population and Samples

Test		Population	Boys	Girls
Lorge-Thorndike Intelligence	$\bar{X}$	106.60	105.95	112.02
	s		14.74	12.15
	N	2605	169	191
Iowa Basic Skills				
Vocabulary	$\bar{X}$	5.53	5.60	5.75
	s		1.39	1.34
	N	2520	181	187
Reading Comprehension	$\bar{X}$	5.44	5.43	5.84
	s		1.60	1.46
	N	2520	181	187
Language Skills	$\bar{X}$	5.24	5.07	5.74
	s		1.43	1.29
	N	2520	181	187
Work-Study Skills	$\bar{X}$	5.46	5.50	5.70
	s		1.31	1.13
	N	2520	181	187
Arithmetic Skills	$\bar{X}$	5.05	5.08	5.24
	s		1.04	.97
	N	2520	179	187
Composite	$\bar{X}$	5.35	5.34	5.65
	s		1.22	1.10
	N	2520	179	185

one week. Subjects could choose the week and the school in which they wanted to report for testing. A one-week session was held at Hawthorne School from June 22 to June 26, and a one-week session was held at Hoyt School from July 13 to July 17. Each 2-hour session consisted of a 72 item "test" composed of mathematics items, a 72 item "test" composed of social studies items, and an activity break between the two of approximately 1/2 hour. The mathematics and the social studies items were given first on alternate days.

The data for the boys were collected in a similar manner except that it was done from mid-October to mid-November. Ninety of the boys who were attending Middle School for sixth grade were tested after school for five consecutive days at Schenk (October 19-23), Sennett (October 26-30), and Orchard Ridge (November 2-6) schools; those 105 elementary school boys who completed the testing were tested on three consecutive Saturday mornings (October 10, 17, and 24) at Franklin, Longfellow, and Randall schools.

The mathematics items were arranged in five 72 item "tests." The order of the items was assigned randomly over the 360 items. Two different random orders were used to collect the data: one for each school for the girls and one for each type of school for the boys. The items were arranged in five test booklets according to the random order. The students responded to the items by marking their chosen response directly on an answer sheet. The answer sheets were read by machine and the responses punched onto data cards. The tests were given by experienced test administrators to groups of approximately 30 subjects each.

Treatment of the Data

The treatment of the data consisted of two main procedures: reliability estimation and factor analysis. The data were analyzed separately for each sample. Hoyt analysis of variance reliability estimates were obtained for each of the 30 concept scores and each of

Table 3
Distribution of Fathers' Occupations

Occupation	Boys	Girls
PROFESSIONAL, TECHNICAL, AND KINDRED WORKERS		
00. Accountant	2	2
01. Architect	1	1
02. Dentist	--	--
03. Engineer	5	8
04. Lawyer, Judge	4	3
05. Clergyman	--	--
06. Doctor	7	4
07. Nurse	--	--
08. Teacher, Professor	18	21
09. Other Professional	16	22
FARMER		
11. Farmer	--	--
MANAGERS, OFFICIALS, PROPRIETORS, EXCEPT FARM		
21. Owner of Business	2	--
22. Manager, Official	12	11
CLERICAL AND KINDRED WORKERS		
31. Bookkeeper	--	--
32. Receptionist	--	--
39. Other Clerical and Kindred Workers	3	5
SALES WORKERS		
49. Salesman	20	15
CRAFTSMEN, FOREMEN, AND KINDRED WORKERS (SKILLED WORKERS)		
51. Craftsman, Skilled Worker	31	17
52. Foreman	2	4
53. Armed Services - Officer	1	1
54. Armed Services - Enlisted Man	1	--
OPERATIVES AND KINDRED WORKERS (SEMI-SKILLED WORKERS)		
61. Truck Driver	10	5
62. Operative in Factory	9	8
69. Other Operative and Kindred Workers	18	23
PRIVATE HOUSEHOLD AND SERVICE WORKERS		
71. Fireman	1	3
72. Policeman	1	--
73. Other Protective Service Worker	--	1
74. Practical Nurse, Nurse's Aide	2	--
75. Private Household Workers	1	--
79. Other Service Workers	14	13
81. Non-Farm Laborer	--	--
82. Farm Laborer	--	--
91. Not presently in labor force	4	8
99. Not ascertained	13	22

the 12 task scores for each group studied. Means and standard deviations for each of the scores were also computed.

Factor Analysis

Developing one item for each of the 12

tasks for each of the 30 selected concepts yields a 12 (tasks) by 30 (concepts) matrix consisting of the score for each of the 360 items, one for each cell of the matrix, for each individual to whom the items are administered. A completely crossed design exists and two types of total scores can be secured from this matrix: a total score for each of the 30 concepts (totalled across tasks) and a total score for each of the 12 tasks (totalled across concepts). Figure 1 is an illustration of such a matrix. Using this design to test concept attainment yields data of a three-dimensional type, if more than one concept and more than one task are included. The three dimensions are concepts, tasks, and individuals. The application of conventional factor analysis procedures to such data presents certain problems. As it has been used in the past, the researcher commonly collapses one dimension of the data, thereby losing information that is possibly very important. For example, common practice would be to use mean scores over the set of individuals to create a two-dimensional concept by task matrix which is then "factored."

Tucker's (1966a, 1966b) three-mode factor analysis has made it possible to factor analyze three-dimensional data without the potential loss of information involved in collapsing a dimension. There are some problems, however, in applying the analysis to data collected using the concept by task design with one item per cell. First, the data for a three-mode system are 0-1 data with a single item per cell; thus, there is a reliability problem with single item variables. Second, the common factors in the system are of major interest and the program to which there is access is for a component type analysis. Third, as in ordinary factor analysis, the question of the number of factors (components) to extract is a difficult question to answer, and this information has to be input into the three-mode program. For these reasons the procedures outlined here were used for factor analyzing the mathematics data collected using the schema for testing level of concept attainment.

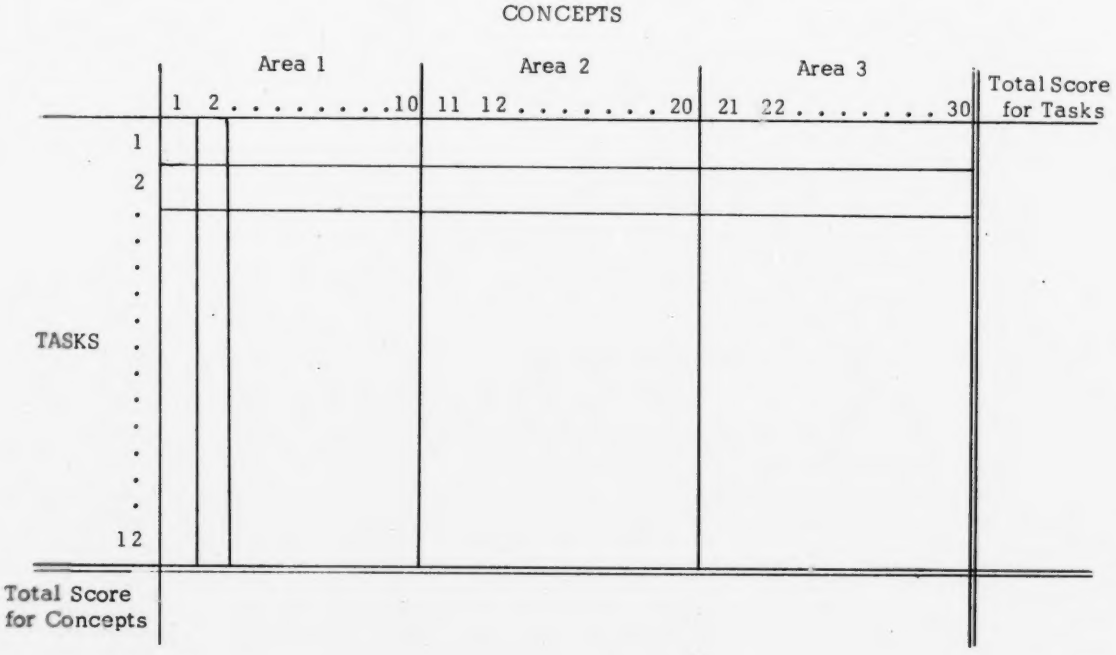

Fig. 1. Item matrix for each individual.

Briefly, the strategy consists of performing conventional factor analyses separately for the concepts and for the tasks to gain some insight into the interrelationships among the variables of a single mode. Tucker's three-mode factor analysis was then used to determine if there are any important interactions among the idealized persons (person factors) and the concepts or tasks.

Conventional Factor Analyses. The original plans called for determining the comparable common factors, separately for the concepts and for the tasks, by using a strategy suggested by Harris and Harris (1970). This strategy is a way to determine those factors that are robust with respect to method—factors which tend to include the same variables across methods. Analyses were obtained using three initial factor methods: Alpha (Kaiser & Caffrey, 1965), Harris R-S^2 (Harris, 1962), and Unrestricted Maximum Likelihood Factor Analysis (UMLFA) (Jöreskog, 1967). These three methods provide a factor solution with a statistical basis with the number of factors determined by a statistical test (UMLFA), and two factor solutions with a psychometric basis: one for a relatively small number of factors (Alpha) and one for a relatively large number of factors (Harris R-S^2). All three of the methods are independent of the scale of the variables. Derived orthogonal solutions were obtained for each of the three initial solutions using the Kaiser normal varimax procedure (Kaiser, 1958), and derived oblique solutions were obtained using the Harris-Kaiser independent cluster solution (Harris & Kaiser, 1964).

The "right number of factors" question is one for which there is still no definitive answer. For matrices which yield about the same number of factors when different methods are used, Harris & Harris (1970) suggest taking the comparable common factors as the substantive results. Doing this, the number of factors can be more or fewer than the number of factors for any single solution. This idea does not seem to be appropriate when the number of common factors obtained using different methods varies considerably, as is the case, for example, with the factoring of the mathematics concepts: for boys and girls respectively 2 each for Alpha, 8 and 7 for Harris R-S^2, and 5 and 3 for UMLFA for the derived orthogonal solutions; the derived oblique solutions yielded 2 each for Alpha, 7 each for Harris R-S^2, and 5 and 3 for UMLFA for boys and girls respectively. These results will be presented more explicitly and discussed in the next section.

Alpha sometimes underfactors, and underfactoring is, according to Kaiser, "an unforgivable sin." Harris R-S^2 extracts a relatively large number of factors (Kaiser calls it deliberate overfactoring); but this is no problem since derived orthogonal common factors retain the important things, get rid of the "garbage," and are in no way substantially affected by doing so (Kaiser, 1970). As an example, for the mathematics concepts, Harris R-S^2 extracted 17 factors initially for the girls but the derived orthogonal solution trimmed these to 7 common factors. Kaiser (1970) advocates this "deliberate overfactoring" but says he wishes oblique transformations were robust to it which they are not. This problem was "solved" by not submitting the initial raw factor matrix to oblique rotation. Instead, the common factors of the derived orthogonal solution were taken as F and used to build R*. The Q obtained from a principal axes decomposition of R* then was submitted for oblique transformation. Thus: derived orthogonal common factors = F; $FF' = R^*$; $R^* = QD^2Q'$ and then this Q is transformed to give an oblique solution. It may be pointed out here that getting derived oblique factors from the initial raw factor matrix or from the Q obtained from R* will not make any difference if the number of initial factors and the number of derived orthogonal common factors is the same; this is the case for the factors obtained for the mathematics concepts and tasks using both Alpha and UMLFA. Incidentally, Kaiser (1970) in the same paper advocates obtaining "Harris factors" as they are model-free. What is named Harris R-S^2 is one of the set of "Harris factors."

This discussion of the number of factors is an important one for this paper since it is necessary to input the number of factors for concepts and the number of factors for tasks into the three-mode program. For these mathematics data the number of factors used was the number of Harris R-S^2 derived oblique common factors. There are several reasons for this: (a) Harris R-S^2 gives as many or more common factors as Alpha or UMLFA and greater specificity should allow any concept-task interactions to be more demonstrable, (b) the Harris R-S^2 solutions "look" better in terms of simple structure and lack of bipolarity, and (c) Henry Kaiser (1970) now advocates that it is the best method.

Three-Mode Factor Analyses. As was mentioned earlier in the paper, three-mode factor analyses (Tucker, 1966a, 1966b) were performed to determine if there are any important interactions among the idealized persons and the concepts or tasks. Three problems were mentioned at that time. Two of them were "solved" by

doing the conventional factor analyses. The common factors in each of the two modes, concepts and tasks, were obtained and the number of factors (components) to input into the three-mode program for the two modes other than individuals was determined. The third problem still remains—the reliability problem with single item variables consisting of 0-1 type data. Also, a fourth problem exists which should perhaps be pointed out at this time. There are some missing data as can be seen in Table 4; instead of 360 items, there are only 353 for boys and 350 for girls. And empty cells cannot be tolerated in a three-mode factor analysis. To alleviate the latter two problems mentioned, single item unreliability and missing data, a three-mode analysis was performed on two different forms of the same data in an attempt to gain insight into the existence of any important concept-task interactions. It might also be pointed out that the existing program has the capacity to handle only a product of 120 for the two modes other than individuals. Thus, we could not analyze our 30 concepts by 12 tasks, as this gives a product of 360. It would have been possible to expand the program's capacity to some extent but it would have been very difficult, if not impossible, to expand it to handle a product of 360.

Conceptually, the 30 concepts were organized by subject matter experts into three areas within the subject matter field. A three-mode analysis was conducted using only three variables for concepts. Each of these variables is a composite of the items for a single task across the ten concepts within a single area. Thus, the input data for this analysis consisted of a 3 (concepts) by 12 (tasks) matrix of 36 entries for each individual. Each entry consisted of the total number correct of ten items (or fewer in the cases of missing data). The number of factors (components) for concepts input for this analysis was taken as three. The number of factors (components) for tasks input for this analysis was the number of derived oblique factors obtained for the Harris R-S^2 method—three for boys and two for girls. This analysis will be referred to as Type I three-mode analysis. Such an analysis should permit any task interactions to be clearly evident, as each task is a separate entry; actually, each task comprises three separate entries, one for each composite concept variable.

A second three-mode analysis, to be referred to as Type II, was conducted using all 30 of the concepts but only three task variables for boys and two for girls. The task variables are composites of the items for a single concept for given tasks. The composites formed for boys are:

Task Variable A - Tasks 2, 3, 4, 5, 10, and 11
Task Variable B - Tasks 1, 6, 8, and 9
Task Variable C - Tasks 7 and 12

The composites formed for girls are:

Task Variable A - Tasks 1, 2, 3, 4, 5, 6, 8, 9, 10, and 11
Task Variable B - Tasks 7 and 12

The formation of the composites was based on the derived oblique factors obtained for the Harris R-S^2 method. A task was assigned to a composite on the basis of its highest factor coefficient. It is realized that this is essentially forming factor scores using a rather undesirable method, but it was felt that since the intercorrelations of the task factors are very high (in fact so high that a reasonable interpretation is that the 12 tasks are all measures of the same latent ability), it would not be too detrimental. Also, it provided a way of forming composites based on experimental results rather than theoretical considerations to allow for greater specificity; an alternative would have been to input only one variable for tasks which would consist of a composite for all 12 of the tasks. Thus, the input data for this Type II three-mode analysis consisted of a 30 (concepts) by 3 or 2 (tasks) matrix of 90 or 60 entries for each individual. The three tasks, and thus 90 entries, are for the analysis of the boys' data and the two tasks and 60 entries are for the girls'. Each entry for the boys consisted of the total number correct of six, four, or two items (or fewer in the cases of missing data) and each entry for the girls consisted of the total number answered correctly of ten or two items (or fewer in the cases of missing data). The number of factors (components) for tasks input for this analysis was taken as three for boys and two for girls. The number of factors (components) for concepts input for this analysis was the number of derived oblique factors obtained for the Harris R-S^2 method—seven for both boys and girls. Such an analysis should permit any concept interactions to be clearly evident since each concept is a separate entry; actually, each concept comprises two or three separate entries, one for each composite task variable. There still may be somewhat of an unreliability problem in this analysis, as some

of the entries consist of the total score for just two items. The results of treating the data in these various ways are presented and discussed in the following section.

III
Results and Discussion

The means, standard deviations, and Hoyt reliability estimates obtained for the data collected during summer and fall of 1970 using the mathematics items developed are presented, separately for boys and girls, for total concept and total task scores. The intercorrelations and factor results for these data are presented and discussed, once again separately for boys and girls.

Reliability Estimates and Test Statistics

Table 4 contains the means, standard deviations, and Hoyt reliability estimates obtained for the data collected during summer and fall, 1970, using the revised items for total concept and total task scores. The data were analyzed separately for the 195 boys and the 196 girls. The key for the task scores appears on the table; the key for the concept scores can be found in Appendix A. For example, Concept 1 is Disjoint Sets, number 2 is Empty Set, number 3 is Equal Sets, etc. In general, the concept scores consist of 12 items each, and the task scores of 30 items each. Exceptions to this are noted in the footnotes.

The mean scores for boys are generally lower than are the mean scores for girls. No conclusions can be drawn from this, however, as the data for the girls were collected in early summer shortly after the school year of their fifth grade had ended and the data for the boys were collected in the fall shortly after the school year of their sixth grade had begun. Thus, it cannot be determined what, if any, of this difference is due to a sex difference and what is due to a time difference and possible forgetting factor. It should also be noted that the scores for Concepts 8, 15, and 22 are based on one more item for boys than they are for girls: Concept 15 has 11 and 10 items for boys and girls, respectively; Concepts 8 and 22 each have 12 items for boys and 11 items for girls, making up the total score. The scores for Tasks 1, 2, and 9 are made up of 30 items for boys but only 29 for girls.

The standard deviations and Hoyt reliability estimates are generally higher for boys than they are for girls.

The reliability estimates are sufficiently high to warrant study of the dimensionality of these selected mathematics concepts and the tasks when using mathematics content. This is a major objective of the CAA Project and is the main purpose for developing these items to measure mathematics concept attainment.

As was mentioned earlier, the subject matter specialists categorized the identified mathematics concepts into three major areas: Set Theory, Division—the inverse of multiplication, and Expressing Relationships. This was done on a theoretical basis. The data could be, and were, analyzed by area for task scores. Instead of a single total task score consisting of the score for that task type item for each of the 30 concepts, three different task scores were obtained for each of the 12 tasks, consisting of the score for that task type item for each of the 10 concepts within a single area. The mean, standard deviation, and Hoyt reliability estimate for each of these 36 scores, 3 areas by 12 tasks, were obtained. Table 5 contains the reliability estimates obtained for task scores by area and for the total across all 30 of the concepts. Spearman-Brown estimates for tripled test lengths (some are given at the bottom of Table 5 for comparison purposes) indicate that the area distinctions are not important ones; the reliability estimates for the total task score are about what would be expected from tripling the length of the test when the single area reliability estimates are of the magnitude that were obtained. The factor results, which will be discussed later, also indicate that the area distinctions

Table 4
Means, Standard Deviations, and Reliabilities for
Mathematics Concept and Task Scores: Boys and Girls

	Concepts[a, b]						Tasks[c]					
	Mean		Standard Dev.		Hoyt Rel.		Mean		Standard Dev.		Hoyt Rel.	
No.	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
1	6.51	7.18	2.20	2.01	.48	.42	18.89	19.34*	5.41	4.13	.81	.71
2	7.09	8.06	2.66	2.32	.67	.61	18.14	19.45*	5.72	5.12	.82	.80
3	6.15	7.25	2.61	2.41	.64	.62	20.16	22.25	5.04	4.00	.80	.73
4	6.99	7.42	2.34	2.34	.55	.60	20.42	22.79	5.03	4.00	.79	.73
5	7.51	8.34	2.33	2.01	.61	.49	18.52	21.05	5.56	4.34	.82	.75
6	6.95+	7.43+	1.99	1.80	.49	.41	16.79	19.44	6.21	5.56	.84	.82
7	5.48	6.36	2.54	2.48	.62	.62	12.63	12.51	4.93	4.61	.73	.70
8	6.82	6.59+	2.49	2.21	.62	.56	16.92	20.40	6.30	5.86	.85	.85
9	5.89	6.10	2.62	2.49	.63	.59	16.94	18.54*	6.02	5.37	.83	.81
10	6.63+	7.43+	2.63	2.04	.71	.58	15.28	17.16	5.33	5.11	.78	.78
11	6.68	8.10	3.00	2.53	.74	.66	11.85**	13.65**	4.49	3.73	.77	.68
12	7.18	8.57	2.66	2.24	.67	.61	12.25	13.51	4.13	3.99	.62	.58
13	5.02	5.48	2.58	2.55	.62	.60						
14	7.69	8.87	2.61	2.47	.69	.73						
15	7.14+	7.28++	2.51	2.14	.69	.68						
16	7.33+	7.64+	2.49	2.27	.71	.66	Key for Tasks:					
17	6.26+	7.19+	2.39	2.28	.62	.63						
18	6.79	7.12	2.94	2.90	.75	.76	1 Given name of attribute, select example.					
19	6.20	6.74	2.69	2.51	.67	.64	2 Given example of attribute, select name.					
20	6.50	7.65	2.55	2.45	.64	.64	3 Given name of concept, select example.					
21	5.66+	5.87+	2.19	2.09	.53	.52	4 Given name of concept, select nonexample.					
22	7.49	7.92+	2.29	1.71	.58	.42	5 Given example of concept, select name.					
23	6.43	7.11	2.31	2.18	.57	.55	6 Given concept, select relevant attribute.					
24	5.21+	6.24+	2.31	2.29	.58	.60	7 Given concept, select irrelevant attribute.					
25	6.65	7.97	2.67	2.39	.65	.62	8 Given definition of concept, select name.					
26	5.65	6.32	2.58	2.16	.65	.50	9 Given name of concept, select definition.					
27	6.35	7.41	2.44	2.33	.61	.61	10 Given concept, select supraordinate concept.					
28	6.83	7.42	2.58	2.02	.65	.44	11 Given concept, select subordinate concept.					
29	7.16	7.84	2.38	2.16	.59	.55	12 Given two concepts, select relationship.					
30	8.55	9.21	2.52	1.93	.71	.64						

[a] The key for the concepts is given in Appendix A.

[b] Scores consist of 12 items each except those marked as follows: + has 11 and ++ has 10.

[c] Scores consist of 30 items each except those marked as follows: * has 29 and ** has 23.

are not important ones.

Factor Analyses

The correlation matrices for the concept scores upon which the factor analyses were based are given in Table 6 for boys and Table 7 for girls. The intercorrelations of the task scores are given in Table 8 for boys and Table 9 for girls.

The intercorrelations of the concept scores are quite consistent in magnitude within the matrix for both boys and girls. The correlations for boys are typically in the .50s and .60s and for girls they are typically in the .40s and .50s. The reliability estimates obtained for the concept scores are generally higher for boys than for girls; typically in the .50s and .60s for boys and the .40s to .60s for girls. Thus, if the correlations were corrected for attenuation they would all be quite high. The lower correla-

Table 5
Reliability Estimates for Task Scores by Area and Total for Girls

	Area			
Task	Set Theory[a]	Division[a]	Expressing Relationships[a]	Total[b]
1	.36	.45+	.51	.71*
2	.53+	.61	.57	.80*
3	.46	.53	.49	.73
4	.41	.55	.49	.73
5	.49	.59	.49	.75
6	.60	.65	.58	.82
7	.42	.54	.33	.70
8	.56	.73	.65	.85
9	.62	.63	.50+	.81*
10	.56	.66	.40	.78
11	.29++	.45+++	.48++	.68**
12	.26	.41	.19	.58

[a] Scores consist of 10 items each except those marked as follows:

+ has 9, ++ has 8, and +++ has 7.

[b] Scores consist of 30 items each except those marked as follows:

* has 29 and ** has 23.

For comparison, these are the Spearman-Brown estimates for tripled test length:

Original	Estimated
.40	.67
.50	.75
.60	.82
.65	.85
.70	.88

Table 6
Intercorrelations of Mathematics Concepts: Boys[a]

Concept	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
2	48																												
3	48	64																											
4	47	58	55																										
5	40	63	55	50																									
6	39	55	64	48	55																								
7	51	54	58	50	57	51																							
8	45	57	63	51	62	64	59																						
9	48	60	52	51	49	40	46	47																					
10	50	66	65	53	66	65	61	66	44																				
11	41	49	55	44	49	53	44	46	35	62																			
12	45	63	57	53	60	58	57	62	43	74	60																		
13	38	48	53	43	52	48	51	50	36	59	53	53																	
14	42	54	57	47	52	55	53	55	44	66	66	67	45																
15	52	62	69	60	61	62	51	60	52	71	60	68	51	72															
16	44	58	56	47	58	57	47	59	44	65	59	64	44	65	62														
17	44	57	58	37	52	50	50	54	41	66	69	60	49	68	63	61													
18	40	55	62	45	56	58	58	57	45	69	60	64	68	60	63	62	63												
19	42	58	60	42	49	59	47	52	46	66	55	61	54	59	62	59	62	66											
20	44	56	58	51	58	56	57	60	44	67	55	67	51	64	65	59	58	67	55										
21	29	47	58	38	44	53	48	49	42	50	46	44	48	49	56	43	47	54	48	45									
22	40	55	48	43	55	51	54	53	45	56	51	61	45	56	56	51	56	53	49	52	43								
23	44	59	62	55	56	54	53	62	45	63	57	63	54	66	69	59	60	63	65	64	51	57							
24	49	52	58	55	52	56	51	53	43	60	53	55	44	51	58	55	49	55	55	56	40	40	54						
25	52	63	58	56	59	56	55	60	42	66	50	65	54	63	65	66	59	63	64	64	45	57	67	54					
26	46	59	64	52	53	50	56	53	55	60	44	61	48	53	63	57	51	55	55	59	48	50	56	50	64				
27	50	65	63	52	51	54	57	63	52	64	43	63	50	61	63	54	55	59	58	58	48	45	56	54	64	66			
28	49	57	56	48	56	54	53	56	42	67	56	59	53	56	61	58	63	59	52	60	48	54	59	56	61	53	57		
29	47	59	60	55	55	57	50	63	43	65	55	65	49	58	66	65	58	63	63	60	47	63	61	54	67	53	60	64	
30	49	69	61	55	65	60	52	60	42	68	56	63	52	59	64	63	57	60	60	61	50	61	61	55	67	55	60	57	67

[a] Decimals have been omitted.

Table 7
Intercorrelations of Mathematics Concepts: Girls[a]

Concept	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
2	49																												
3	49	63																											
4	53	60	51																										
5	41	52	51	47																									
6	40	47	52	41	35																								
7	33	47	50	36	51	40																							
8	49	56	55	53	59	43	51																						
9	49	51	50	45	40	36	36	51																					
10	53	61	58	55	55	60	46	55	48																				
11	48	54	55	48	48	40	46	54	37	49																			
12	55	63	52	50	43	51	43	53	41	59	56																		
13	42	45	51	33	39	34	38	41	46	50	43	45																	
14	57	63	56	59	56	41	42	58	44	56	62	53	40																
15	55	67	55	55	56	44	47	60	42	62	61	57	48	69															
16	41	50	50	43	47	44	42	51	51	60	44	53	56	49	47														
17	40	50	51	39	43	38	38	51	43	42	64	41	33	61	56	42													
18	51	56	56	49	51	44	43	57	48	61	50	60	56	54	56	53	47												
19	48	55	47	48	47	44	40	55	48	59	45	59	53	54	54	52	39	56											
20	45	54	48	56	45	45	42	52	50	56	47	56	42	53	60	53	40	51	55										
21	42	45	45	37	34	33	28	35	45	49	35	42	36	45	37	42	26	42	35	42									
22	37	48	40	40	37	29	46	49	40	44	42	47	37	41	45	41	33	44	38	45	38								
23	46	56	50	57	45	46	37	47	49	54	42	44	37	53	50	45	41	44	48	53	34	40							
24	44	48	55	35	49	43	37	53	45	47	49	47	42	43	55	44	41	43	42	45	36	37	41						
25	52	59	51	50	51	43	44	51	47	53	40	48	41	48	55	47	42	51	48	50	33	38	51	52					
26	49	52	52	55	43	39	39	53	51	51	41	50	46	49	51	45	42	55	47	51	39	42	52	42	59				
27	49	60	60	49	53	45	41	55	46	59	51	53	47	55	58	53	53	55	48	54	43	41	54	50	58	57			
28	48	59	56	52	43	44	47	52	47	53	47	48	47	51	53	51	43	51	45	54	44	46	51	40	46	51	55		
29	43	62	59	51	47	53	44	60	43	60	50	61	45	57	62	53	47	56	54	53	37	44	53	50	44	45	63	57	
30	41	58	54	58	57	44	48	55	39	57	55	56	38	55	60	49	47	50	44	49	31	43	46	41	47	49	51	44	54

[a] Decimals have been omitted.

Table 8
Intercorrelations of Mathematics Tasks: Boys[a]

Task	1	2	3	4	5	6	7	8	9	10	11
2	83										
3	81	84									
4	78	78	76								
5	84	83	83	79							
6	82	82	81	75	81						
7	59	62	59	59	64	68					
8	84	83	81	76	83	83	69				
9	83	80	80	73	80	83	65	86			
10	78	78	79	74	80	80	65	79	77		
11	76	78	77	73	77	78	66	79	75	75	
12	70	70	68	62	69	73	64	70	70	65	69

[a] Decimals have been omitted.

Table 9
Intercorrelations of Mathematics Tasks: Girls[a]

Task	1	2	3	4	5	6	7	8	9	10	11
2	84										
3	82	78									
4	74	74	73								
5	76	79	78	74							
6	77	79	79	73	77						
7	51	53	55	48	55	61					
8	78	82	80	78	80	84	56				
9	78	80	78	77	77	78	54	83			
10	74	76	73	71	74	77	54	78	71		
11	72	76	73	67	76	71	51	75	72	72	
12	59	64	67	57	64	65	59	68	66	66	62

[a] Decimals have been omitted.

tions obtained are almost wholly associated with the concept scores which have low reliability estimates.

The intercorrelations of the task scores are quite consistent in magnitude for boys and girls. They are in the .70s and .80s for both boys and girls except for Tasks 7 and 12. The correlations of Task 7 with the remaining tasks are in the .50s for girls and .60s for boys; the correlations of Task 12 with the remaining tasks are in the .60s for girls and .60s to low .70s for boys. Once again, it is interesting to look at the reliability estimates for the task scores. They are generally higher for boys and are typically in the .70s and .80s. Task 12, however, has a reliability estimate of just .58 for girls and .62 for boys; Task 7 is in the low .70s for both boys and girls. Thus, as with the concepts, if the correlations were corrected for attenuation, they would almost all be very high.

Conventional Factor Analyses

The numbers of factors obtained for the

initial solutions and for the derived solutions, orthogonal and oblique, are given in Tables 10 and 11 according to the numbers of common, specific, and null factors. A common factor is defined as one having at least two variables with coefficients greater than .30 (absolute); a specific factor has only one coefficient greater than .30 (absolute); and a null factor does not have any coefficients greater than .30 (absolute). The factors rotated for the derived oblique solutions were the orthogonal common factors obtained for that method. For this purpose a common factor was defined as one having at least two variables with coefficients greater than .300 (absolute).

Interpretation of Factor Results for Concept Scores. The factor results for the concepts can be interpreted at two levels. One level is a general one. A reasonable interpretation is that all 30 of the concepts are measures of a single functional relationship existing among the concepts; this holds for both boys and girls. At least four things lead to such an interpretation. First, the intercorrelations of the 30 concepts are all quite uniform. They would probably fit a Spearman pattern fairly well; this indicates a single common factor. The correlations, if corrected for attenuation, would all be quite high. The eigenvalues of the correlation matrices obtained for

Table 10
Numbers of Initial and Derived Factors for Concept Scores: Boys and Girls

Factor Method	Initial Factors		Derived Orthogonal Factors						Derived Oblique Factors					
			Common		Specific		Null		Common		Specific		Null	
	B	G	B	G	B	G	B	G	B	G	B	G	B	G
Alpha	2	2	2	2	0	0	0	0	2	2	0	0	0	0
Harris R-S^2	18	17	8	7	3	3	7	7	7	7	1	0	0	0
UMLFA	5	3	5	3	0	0	0	0	5	3	0	0	0	0

Table 11
Numbers of Initial and Derived Factors for Task Scores: Boys and Girls

Factor Method	Initial Factors		Derived Orthogonal Factors						Derived Oblique Factors					
			Common		Specific		Null		Common		Specific		Null	
	B	G	B	G	B	G	B	G	B	G	B	G	B	G
Alpha	1	1	1	1	0	0	0	0	1	1	0	0	0	0
Harris R-S^2	5	6	3	2	0	0	2	4	3	2	0	0	0	0
UMLFA	1	2	1	2	0	0	0	0	1	2	0	0	0	0

The derived orthogonal common factor results can be found in Appendices B-E; the derived oblique common factor results are presented in Tables 12-15. Only coefficients greater than .30 (absolute) are included. The order of the factors for each solution is arbitrary. The intercorrelations of the factors are included in the tables for the oblique solutions.

both boys and girls are characterized by the first one being very large followed by a great drop in magnitude to the next ones which diminish very gradually. Finally, the oblique factor intercorrelations are uniformly quite high, indicating only one second-order factor. Such an interpretation is reasonable in terms of past studies, also. In the literature for

Table 12
Oblique Common Factor Results for Mathematics Concepts: Boys[a]

	Alpha		Harris R-S[2]							UMLFA				
Concept	A-1	A-2	H-1	H-2	H-3	H-4	H-5	H-6	H-7	U-1	U-2	U-3	U-4	U-5
Area: Set Theory														
1 Disjoint Sets	75					89				38				38
2 Empty Set	77								66	34			50	
3 Equal Sets	53						72			95				
4 Equivalent Sets	94					51				54				
5 Line	43	33							113				104	
6 Parallel Lines		59					77						32	
7 Plane	60							-49		32			32	
8 Point	46	32					48		46				44	
9 Subset	105	-41								87				
10 Subtraction		79							47				36	
Area: Division														
11 Denominator	-48	118		97	-37						106			-40
12 Division		73			43				46					62
13 Factor		63	80									78		
14 Fraction		94		86	33						70		-35	65
15 Mixed Fraction	31	54		39			32			44	37			33
16 Multiplication		76						49	31					67
17 Numerator	-34	110		83							68			
18 Product		97	60									78		
19 Quotient		79						88	-31			31		60
20 Remainder		63			36									56
Area: Expressing Relationships														
21 Average		48					79			71		31		-40
22 Graph		54							65				52	
23 Measurement		67						42						53
24 Open Sentence	44					60				34				
25 Place Holder	32	50			43			60						127
26 Place Value	74				100					54				66
27 Solution Set	72				75					45				86
28 Standard Unit		62				32								
29 Statement		59						74						65
30 Weight		52						35	70				60	
Intercorrelations of factors 2	91		80							79				
3			80	84						79	81			
4			74	79	88					89	82	80		
5			83	83	88	83				91	87	83	92	
6			81	89	90	84	87							
7			81	86	90	88	88	91						

[a] Includes those variables which have coefficients greater than .30 (absolute). Decimals have been omitted.

Table 13
Oblique Common Factor Results for Mathematics Concepts: Girls[a]

Concept	Alpha		Harris R-S[2]							UMLFA		
	A-1	A-2	H-1	H-2	H-3	H-4	H-5	H-6	H-7	U-1	U-2	U-3
Area: Set Theory												
1 Disjoint Sets	64				58							35
2 Empty Set	43	39			40							39
3 Equal Sets	47									56		
4 Equivalent Sets	42				92							108
5 Line		63							88		36	
6 Parallel Lines	48					95				60		
7 Plane		46					69			47		
8 Point		57							32	37	35	
9 Subset	99	-34			57					75		
10 Subtraction	80					53			35	75		
Area: Division												
11 Denominator		98		85							97	
12 Division	56					32				58		
13 Factor	88		93							120		-50
14 Fraction		75		62							64	39
15 Mixed Fraction		80		37							57	
16 Multiplication	86		58							103		
17 Numerator		92		95							92	
18 Product	70		53							81		
19 Quotient	72		51							73		
20 Remainder	71				48					45		38
Area: Expressing Relationships												
21 Average	88	-34								65		
22 Graph	46						69			44		
23 Measurement	58				80							67
24 Open Sentence	39							66		61		
25 Place Holder	57				74			31		47		34
26 Place Value	77				94					48		44
27 Solution Set	52				35					56		
28 Standard Unit	70				39		38					
29 Statement	39	41				46						
30 Weight		78							45		41	31
Intercorrelations of factors 2	91		74							87		
3			84	82						89	83	
4			82	78	86							
5			80	79	83	80						
6			75	72	75	72	71					
7			78	85	86	82	84	74				

[a] Includes those variables which have coefficients greater than .30 (absolute).

Decimals have been omitted.

Table 14
Oblique Common Factor Results for Mathematics Tasks: Boys[a]

Task	Alpha	Harris R-S[2]			UMLFA
	A-1	H-1	H-2	H-3	U-1
1 Given name of attribute, select example.	90	47	69		91
2 Given example of attribute, select name.	91	95			91
3 Given name of concept, select example.	90	98			90
4 Given name of concept, select nonexample.	84	110			84
5 Given example of concept, select name.	91	89			91
6 Given concept, select relevant attribute.	91		45		91
7 Given concept, select irrelevant attribute.	72			91	72
8 Given definition of concept, select name.	92		81		92
9 Given name of concept, select definition.	89		111		90
10 Given concept, select supraordinate concept.	87	80			87
11 Given concept, select subordinate concept.	86	63		39	85
12 Given two concepts, select relationship.	78			49	77
Intercorrelations of factors: 2		98			
3		91	93		

[a] Includes those variables which have coefficients greater than .30 (absolute).

Decimals have been omitted.

Table 15
Oblique Common Factor Results for Mathematics Tasks: Girls[a]

Task	Alpha	Harris R-S[2]		UMLFA	
	A-1	H-1	H-2	U-1	U-2
1 Given name of attribute, select example.	87	112		125	-39
2 Given example of attribute, select name.	89	100		104	
3 Given name of concept, select example.	88	80		84	
4 Given name of concept, select nonexample.	82	97		88	
5 Given example of concept, select name.	88	77		73	
6 Given concept, select relevant attribute.	89	62		59	32
7 Given concept, select irrelevant attribute.	63		83		85
8 Given definition of concept, select name.	91	77		67	
9 Given name of concept, select definition.	88	87		80	
10 Given concept, select supraordinate concept.	85	58		55	32
11 Given concept, select subordinate concept.	83	70		67	
12 Given two concepts, select relationship.	75		75		85
Intercorrelations of factors: 2		90		91	

[a] Includes those variables which have coefficients greater than .30 (absolute).

Decimals have been omitted.

factor studies that include measures of achievement, the results typically indicate that achievement measures are found on a single factor. We have here achievement measures for a single subject matter field which, conceptually at least, should be even more closely related than achievement measures from several different areas of study.

The other level at which the factor results can be interpreted is a more specific one. The derived orthogonal factors are not very meaningful; they are not very interpretable psychologically. As can be seen from Tables 12 and 13, the oblique factors are very highly correlated; thus, imposing the restriction of orthogonality on them gives results that are not very meaningful. Many of the variables are of complexity 2, 3, and even higher in the orthogonal solutions. For example, for the two factors of the Alpha solutions, almost all of the concepts have coefficients greater than .30 on both of the factors; for the UMLFA solutions, most of the variables are of complexity 3 for the three factors obtained for boys and many are of complexity 3, 4, and in one case, 5, for the five factors obtained for the girls. Even for the greater number of factors for the Harris R-S^2 solutions, there are still a number of concept variables of complexity 2 and 3. Thus, at a more specific level, the only solutions which it makes any sense to interpret are the oblique ones. It must be remembered, however, that the correlations of these factors are all quite high.

For matrices which yield about the same number of factors when different methods are used, Harris and Harris (1970) suggest taking the comparable common factors, those that are robust over method, as the substantive results. This idea does not seem to be appropriate when the number of common factors obtained using different methods varies considerably, as is the case with the factoring of these mathematics concepts: for boys and girls respectively, 2 each for Alpha, 7 each for Harris R-S^2, and 5 and 3 for UMLFA. Thus, it seems the only appropriate thing would be to look at the results for each method individually.

The results for the boys are given in Table 12. For these mathematical concepts, the Alpha results are the easiest to interpret. The two factors replicate factors found in many other studies. Factor A-1 is an "abstract concepts" factor. It includes such concepts as Subset, Equivalent Sets, Solution Set, Place Value, etc. All are abstract concepts. Note that the Set concepts have large coefficients on this factor as do those concepts dealing with abstract relationships. The second Alpha factor, A-2, includes concrete or operational concepts. All of the Division area concepts appear on this factor, as do all of the operational concepts. More familiar or concrete concepts from geometry—Parallel Lines, Line, and Point—appear on this factor to some extent. Line and Point have larger coefficients on A-1, however. The items developed to measure these concepts were oriented more toward the abstract than the concrete. For example, the definition of Line is: A line has no end but does have a length and can be drawn using two points.

The UMLFA method gives somewhat the same results. U-1 is the same abstract concepts factor as is A-1. The Set concepts still form the basis of this factor. U-2 is a fractions concept since it includes Denominator, Numerator, Fraction, and Mixed Fraction, and nothing else. U-3 is curious. It first seemed to be a multiplication factor since it includes Factor and Product, and, to some extent, Quotient and Average. The curious thing is that the concepts Multiplication and Division do not appear on the factor. Thus, it includes elements that are operated upon but not the operations themselves. U-4 seems to be a factor dealing with geometric and measurement concepts. It includes such things as Line, Point, and Graph which indicate geometric relationships, and Weight which is a kind of measurement. The concept of Measurement itself does not appear on this factor, however. The appearance of Empty Set and Subtraction on this factor is hard to explain. U-5 is another very difficult factor to explain. Large coefficients for Place Holder, Solution Set, Statement, and Measurement indicate a factor for mathematical sentences and their solution. Certainly the appearance of many of the Division area concepts on this factor adds to this interpretation. The appearance of Disjoint Sets (with a small coefficient only) and the failure of Open Sentence to be included are unexplainable, however.

The Harris R-S^2 solution is, in general, much more difficult to interpret than the others. H-1 is a doublet factor for the two concepts Factor and Product. H-2 is the same Fractions factor as U-2. From H-3 on, the factors are more difficult to interpret. Place Value and Solution Set clearly indicate that H-3 is a numeric factor with relatively small coefficients for Place Holder, Division, Remainder, and Fraction. There seems to be no rationale for the appearance of Disjoint Sets, Equivalent Sets, Open Sentence, and Standard Unit on H-4. There are no immediately obvious relationships for the concepts which appear on H-5. Looking closer, the three main concepts

on the factor—Equal Sets, Parallel Lines, and Average—could perhaps indicate some kind of "common ground" type of relationship. Point and Mixed Fraction which appear with somewhat smaller coefficients do not fit into such an interpretation, however. Once again, there seems to be no good interpretation for H-6 which includes Quotient, Multiplication, Statement, Place Holder, Measurement, and Weight. Similarly, the concepts which appear on H-7—Line, Point, Empty Set, Subtraction, Graph, Weight, Division, and Multiplication—do not seem to be related conceptually. It is interesting to note that the concepts which are names of operations and those which are elements operated upon do not appear on the same factors; see Factors H-1, H-6, and H-7. This was the case for Multiplication in the UMLFA solution, but Division and its elements did appear on the same factor, U-5. This does not hold for the girls as will be seen later.

The results for the girls are given in Table 13 and will be interpreted here. Once again, the Alpha results are the easiest to interpret. The two Alpha factors parallel the two obtained for the boys but with some rather striking differences. A-1 seems to be the abstract concepts factor and A-2 the concrete or operational concepts factor. However, the abstract concepts factor includes many of the 30 concepts and is much more inclusive than it is for boys; for boys, the concrete or operational concepts factor included more concepts than the abstract one. Unlike the boys, for the girls the Division area concepts split on the two factors, those for fractions appearing on A-2 and the remaining ones on A-1; also unlike the boys, the concepts for the area of Expressing Relationships almost all appear on A-1. Perhaps at this fifth grade level more mathematical concepts are at an abstract level for girls but at a concrete or operational level for boys.

The UMLFA method yielded three factors for girls as compared to five for boys. U-1 seems to be essentially the same abstract concepts factor as A-1. It is still very broad, including many of the concepts in each of the three areas. The main difference between A-1 and U-1 is that some of the Set concepts and Measurement split off and form another factor, U-3. U-2 is basically the Fractions factor.

As with the boys, Harris R-S[2] results are much more difficult to interpret than the others. The first three factors are quite clear; the remaining ones are not so clear. H-1 appears to be a multiplication factor including the concepts Factor, Product, Multiplication, and Quotient. The concepts for both the operation and the elements operated upon appear on this factor; this type of result is unlike the results for boys in this respect. It is curious that Quotient appears on this factor but not Remainder or Division. H-2 is the factor for the fractions concepts. H-3 appears to be the general abstract concepts factor, though it is not as inclusive as it is for the Alpha and UMLFA solutions. It includes all of the Set concepts except Equal Sets and most of the abstract concepts from the Expressing Relationships area. There seems to be no rationale for explaining H-4. Two of the three operations studied, Subtraction and Division, appear here, but they seem unrelated to the other two concepts, Parallel Lines and Statement. H-5 may be interpreted as including concepts that deal with less familiar geometric concepts since the two main concepts on the factor are Plane and Graph. H-6 is a doublet for Open Sentence and Place Holder. The relationship here seems to be that a place holder is used in an open sentence. H-7 is a curious factor with Point and Line both appearing on it, but with coefficients of quite different magnitude. The other two concepts that appear on H-7 are Weight and Subtraction.

It is interesting to point out that the factor for "abstract concepts" appeared for all three of the methods for the girls but only for the Alpha solution for the boys.

It is evident from the factor results that the three area distinctions are not functional; thus, the hypothesis that mathematical concepts are functionally related according to the three conceptually-determined major content areas must be rejected.

A word of caution. Too much emphasis should not be placed on the distinctions just discussed, as the intercorrelations of the factors are quite high. The abstract and concrete factors of the Alpha solution are correlated .91 for both boys and girls. There are only four concepts that are of complexity 2 for girls and 7 for boys. Of these, two are bipolar for girls and three are bipolar for boys. As one would expect, as the results become more specific (more factors) the factors are less correlated. However, for the seven factors of the Harris R-S[2] solution, the correlations are in the .70s and .80s for girls and the high .70s to low .90s for boys; these correlations are quite high, especially considering that there are very few variables on many of the factors.

It may be well to insert a reminder here that the orthogonal solutions are not very meaningful psychologically, since the complexity is greater than 1 for most of the concepts; most of the concepts appear on more than one factor.

GPO 827-030-5

The most interesting aspect of studying these mathematics concepts is yet to come: the study of the relationships of selected mathematics concepts with selected concepts from the other three subject matter fields being studied (language arts, science, and social studies). This is Step 4 of the objectives of the CAA Project as stated on Page 1.

Interpretation of Factor Results for Task Scores. As with the concepts, the factor results for the tasks can be interpreted at two levels. One level is a general one; all 12 of the tasks are measures of a single underlying ability or latent trait. This seems to be the most reasonable interpretation for the tasks since the intercorrelations of the oblique factors are very high when more than one factor is yielded. All of the reasons for a general interpretation for the concepts apply for the interpretation of the tasks: (a) the intercorrelations are all quite high and quite uniform—they would fit a Spearman pattern fairly well, (b) the correlations corrected for attenuation would all be very high, (c) the eigenvalues of the correlation matrices are characterized by the first one being very large followed by a great drop in magnitude to the next ones, and (d) the factor intercorrelations are uniformly very high, indicating only one second-order factor.

At a more specific level, only the oblique factor results are psychologically meaningful. These results are given in Table 14 for boys and Table 15 for girls.

For the boys, both Alpha and UMLFA yielded only one common factor. The factor coefficients for both of these solutions are uniformly very high. The Harris R-S^2 solution yielded three factors, but H-1 and H-2 are correlated .98; it is rather senseless to try to make any distinction between these two factors. H-3 is correlated .91 and .93 with H-1 and H-2 respectively. It basically includes Task 7 and Task 12. Both of these tasks go beyond the characteristics of the concept itself and involve relationships with other concepts. Task 12 does this directly; Task 7 does it by requiring that the student distinguish between attributes that are necessary for an exemplar to be identified as an exemplar of that particular concept (relevant attributes) and those that are an attribute of the concept but are not necessary to identify it as an exemplar of that particular concept (irrelevant attributes). For example, a relevant attribute of Equivalent Sets is that they contain the same number of members; what those members are is irrelevant. What the members are is a relevant attribute for Equal Sets, however; the members of two or more equal sets must be identical. Irrelevant attributes often identify concepts that are conceptually subordinate to a given concept. In this sense, Task 7 involves relationships with other concepts. Task 6 is essentially the reverse of Task 7, however. It requires selecting a relevant attribute from irrelevant ones while Task 7 requires selecting an irrelevant attribute from relevant ones.

For the girls, Alpha yielded just one factor while Harris R-S^2 and UMLFA both yielded two factors. They are essentially the same factors as the two somewhat different ones of the Harris R-S^2 solution for the boys. H-1 and U-1 include all of the task variables except Task 7 and Task 12. These two tasks comprise H-2 and are the main variables on U-2.

As with the concepts, the most interesting aspect of studying these tasks using mathematics content will be to see the relationships to these same tasks when language arts, social studies, and science concepts are used as content.

Three-Mode Factor Analyses

As was discussed earlier, a three-mode factor analysis was performed on two different forms of the same data to gain insight into the existence of any important concept-task interactions for the idealized persons. Performing conventional factor analyses on the two modes, concepts and tasks, separately is essentially hypothesizing that there are no interactions. The three-mode analyses were performed to determine whether this hypothesis is a tenable one.

The Type I three-mode analysis is the analysis of the 12 tasks and the three composite concept variables; Type II is the analysis of the two (girls) or three (boys) composite task variables and the 30 concepts. Type I was performed to permit maximum task interactions to be evident; Type II to permit maximum concept interactions.

The core matrix obtained for each analysis is the only piece of the three-mode analysis of interest here since it contains the idealized person components by task components by concept components. Hence, it is in this matrix that any interactions are seen. The core matrices obtained for Type I and Type II analyses are presented in Table 16 for boys and in Table 17 for girls. Only those idealized person (core) components that have one or more coefficients greater than .50 (absolute) are included in the tables; the number of core components obtained

in each of the analyses was equal to the product of the number of components for the two modes other than individuals. The variables comprising the task components are given in footnotes on each of the tables. The variables comprising the Type I concept components are the ten concepts in each of the three areas. The concept components for the Type II analyses bear some resemblance to the Harris R-S[2] factors which were the basis for the number of components to be extracted but they are much more specific. Most of them have only two or three variables with coefficients greater than .30 (absolute).

Both Type I and Type II analyses for the boys indicate that there is only one idealized person type—there is just one major core component. As indicated by the Type I analysis, persons respond similarly to the concepts of the three different areas; the Type II analysis indicates some slight differentiation among the concepts. Both analyses indicate that a person who scores well tends to do less well on Tasks 7 and 12; he does poorly on Task 7 for Area 2 concepts and average on concept component 4 in the Type II analysis, which is a component comprised essentially of concept variable 1 (Disjoint Sets). A person with low scores for core component 1 would tend to perform better on Tasks 7 and 12 than on the remaining tasks. In the Type I analysis there are no other coefficients greater than .75 (absolute). Minor variations in response patterns for the idealized persons can be seen in Table 16. In the Type II analysis there is just one other coefficient that is greater than .75 (absolute). Idealized person type 3 tends to do less well on concept component 7; this is essentially the same as the H-1 factor for task component 3 which is comprised of Tasks 7 and 12. Minor variations in response patterns for the idealized persons for the Type II analysis can be seen in Table 16.

The three-mode results for the girls are essentially the same as for the boys; there is just one major core component indicating just one idealized person type. As with the boys, a person with high scores on this core component does less well on Tasks 7 and 12 than on the other tasks. She responds similarly to the concepts of the three different areas with some slight variations among the concepts as indicated by the Type II analysis. In the Type I analysis there is just one coefficient greater than .75 (absolute); idealized person type 2 tends to do poorly on Area 2 concepts for Tasks 7 and 12. There is one other coefficient greater than .75 (absolute) in the Type II analysis; idealized person type 3 does poorly on concept component 5 which is essentially concept variable 30 (Weight) for Tasks 7 and 12. All other slight variations can be seen in Table 17. For the Type II analysis these other slight variations in response patterns are limited to task component 2 which is comprised of Tasks 7 and 12.

The results of the three-mode factor analyses support the hypothesis that there are no important concept-task interactions for the idealized persons. Thus it is reasonable to regard these two modes as independent.

Table 16
Three-Mode Core Results: Boys

Idealized Persons	Task Components[a]	Type I Concept Components: Area 1	Area 2	Area 3
1	1	2.43	2.41	2.33
	2	.78	-.71	.52
	3	.64	.52	.64
2	1	.27	.18	.01
	2	-.25	.42	-.71
	3	.16	-.29	-.35
3	1	.61	-.08	-.63
	2	.24	-.14	.20
	3	.01	-.11	-.26
4	1	-.03	-.16	-.08
	2	-.26	.08	-.11
	3	.58	.25	.10

Idealized Persons	Task Components[a]	Type II Concept Components: 1	2	3	4	5	6	7
1	1	.72	1.61	.97	.58	2.41	.89	1.11
	2	.61	1.49	1.07	.87	2.35	.77	1.07
	3	.27	.69	.43	-.04	1.42	.71	.60
2	1	-.19	-.10	.13	-.26	.57	.19	-.14
	2	.03	-.38	-.06	-.02	.30	.03	-.06
	3	-.49	-.56	.46	-.44	-.23	-.06	-.07
3	1	.14	.09	-.01	-.13	.09	-.02	-.24
	2	.18	-.19	-.06	.21	.18	.21	-.14
	3	.02	-.22	.50	.06	-.16	.13	-.78
4	1	-.05	.21	-.22	.01	.06	-.23	-.14
	2	-.08	.04	.11	.07	.02	-.22	-.35
	3	-.25	-.07	-.26	.55	.43	.15	-.19
5	1	-.06	-.11	.05	-.03	-.11	.12	.06
	2	.11	-.12	.18	-.10	-.10	-.02	.11
	3	-.08	-.52	-.07	-.01	-.01	.28	-.32

[a] Variables comprising task components:

Type I: 1 - Tasks 1 - 6, and 8 - 11
2 - Task 7
3 - Task 12

Type II: 1 - Tasks 2 - 5, 10, 11
2 - Tasks 1, 6, 8, 9
3 - Tasks 7, 12

Table 17
Three-Mode Core Results: Girls

Idealized Persons	Task Components[a]	Type I Concept Components: Area 1	Area 2	Area 3
1	1	2.33	2.27	2.32
	2	.93	.58	.76
2	1	.33	.17	.02
	2	-.33	-.83	-.58
3	1	-.56	.50	.14
	2	-.14	-.12	.09

Idealized Persons	Task Components[a]	Type II Concept Components: 1	2	3	4	5	6	7
1	1	2.12	1.46	1.08	1.56	.52	1.62	1.24
	2	1.16	.21	.54	.69	.23	.27	.89
2	1	.27	.22	.11	-.07	-.21	.14	.34
	2	-.41	-.46	-.34	-.63	-.38	-.65	.01
3	1	-.34	.15	-.13	.14	.14	.17	.06
	2	-.38	-.05	-.09	.26	-.80	.35	.47
4	1	.00	-.07	-.23	.08	.03	.15	.11
	2	-.18	.58	.36	-.38	.02	-.11	-.02
5	1	-.23	.00	.15	-.08	.06	.13	.15
	2	-.31	-.34	.66	-.14	.17	.24	.11
6	1	.00	-.10	.16	-.10	-.14	-.04	-.12
	2	.07	-.04	.38	.08	-.14	-.56	.28

[a] Variables comprising task components:

Types I and II:

Component 1 - Tasks 1, 2, 3, 4, 5, 6, 8, 9, 10, and 11
Component 2 - Tasks 7, and 12

IV
Summary and Conclusions

The primary objective of the project entitled "A Structure of Concept Attainment Abilities" is to formulate one or more models or structures of concept attainment abilities, and to assess their consistency with actual data. This paper contains a report of the factor analytic study of the content and task dimensions of the mathematics items.

Mathematics items to measure concept attainment were developed using a completely crossed design utilizing 30 concepts and 12 tasks. The mathematics items were administered during the summer of 1970 to 196 girls who had just completed the fifth grade and during the fall of 1970 to 195 boys who had just begun the sixth grade.

Two types of total scores were secured from the students' responses to the mathematics items—a total score for each of the 30 concepts (totalled across tasks) and a total score for each of the 12 tasks (totalled across concepts). Means, standard deviations, and Hoyt reliability estimates were obtained for each of the 30 concept scores and each of the 12 task scores for each of the groups studied.

Conventional factor analyses were performed separately on the intercorrelation matrices obtained for the concepts and for the tasks for the boys and the girls. Analyses were obtained using three initial factor methods: Alpha (Kaiser & Caffrey, 1965), Harris R-S^2 (Harris, 1962), and Unrestricted Maximum Likelihood Factor Analysis (Jöreskog, 1967). Derived orthogonal solutions were obtained for each of the three initial solutions using the Kaiser normal varimax procedure (Kaiser, 1958) and derived oblique solutions were obtained using the Harris-Kaiser independent cluster solution (Harris & Kaiser, 1964).

Three-mode factor analysis (Tucker, 1966a, 1966b) was performed on two different forms of the same data to determine whether there are any important interactions among the idealized persons and the concepts and/or tasks.

The conventional factor results for the concepts yielded two or more orthogonal factors for the various methods. The concept variables are almost all of complexity 2, 3, and even greater on these factors, however. The oblique results tend to yield simple structures but the oblique factors are very highly correlated; thus, the main conclusion is that all 30 of the concepts are measures of a single functional relationship existing among the concepts. This holds for both boys and girls.

As with the concepts, the most reasonable interpretation for the tasks is that all 12 of the tasks are measures of a single underlying ability or latent trait. The intercorrelations of the oblique factors are extremely high when more than one factor is yielded.

The results of the three-mode factor analyses support the hypothesis that there are no important concept-task interactions for the idealized persons. Thus, it is reasonable to regard these two modes as being independent.

The most interesting aspect of studying these mathematics items will be to see how they are related to concepts from three other subject matter fields (language arts, science, and social studies) and to general cognitive abilities. The data for such a study will be collected during summer, 1971. Even though a reasonable interpretation is that there is only a single common factor for the 30 concepts, the most specific results obtained were used to determine what mathematics concepts to include in the summer, 1971, study. This should permit maximal demonstration of relationships with concepts from other subject matter fields. The two concepts with the highest coefficients on each of the Harris R-S^2 factors for both the boys and girls were selected. On this basis a total of 18 mathematics concepts were selected for further study. These concepts are: Disjoint Sets, Equivalent Sets, Line, Parallel Lines, Plane, Subtraction,

Denominator, Factor, Fraction, Product, Quotient, Average, Graph, Open Sentence, Place Value, Solution Set, Statement, and Weight. Even though the most reasonable interpretation for the tasks is that there is a single common factor, all 12 of the tasks will be included in the summer, 1971, study in order to have a reliable concept score (totalled across the 12 tasks for a single concept).

References

Bourne, L. E., Jr. Human conceptual behavior. Boston: Allyn & Bacon, 1966.

Frayer, D. A., Fredrick, W. C., & Klausmeier, H. J. A schema for testing the level of concept mastery. Madison: Wisconsin Research and Development Center for Cognitive Learning, Working Paper No. 16, 1969.

Harris, C. W. Some Rao-Guttman relationships. Psychometrika, 1962, 27, 247-263.

Harris, C. W., & Kaiser, H. F. Oblique factor analytic solutions by orthogonal transformations. Psychometrika, 1964, 29, 347-362.

Harris, M. L., & Harris, C. W. A factor analytic interpretation strategy. Wisconsin Research and Development Center for Cognitive Learning, Technical Report No. 115, 1970. (Also published in Educational and Psychological Measurement, 1971, 31, 589-606.)

Harris, M. L., Harris, C. W., Frayer, D. A., & Quilling, M. R. A structure of concept attainment abilities: The problem and strategies for attacking it. Wisconsin Research and Development Center for Cognitive Learning, Theoretical Paper No. 32, in press.

Harris, M. L., & Romberg, T. A. Measuring mathematics concept attainment: Boys and girls. Wisconsin Research and Development Center for Cognitive Learning, Technical Report No. 195, in press.

Jöreskog, K. G. Some contributions to maximum likelihood factor analysis. Psychometrika, 1967, 32, 443-482.

Kaiser, H. F. The varimax criterion for analytic rotation in factor analysis. Psychometrika, 1958, 23, 187-200.

Kaiser, H. F. A second generation Little Jiffy. Psychometrika, 1970, 35, 401-415.

Kaiser, H. F., & Caffrey, J. Alpha factor analysis. Psychometrika, 1965, 30, 1-14.

Klausmeier, H. J., Harris, C. W., Davis, J. K., Schwenn, E., & Frayer, D. A. Strategies and cognitive processes in concept learning. The University of Wisconsin, Cooperative Research Project No. 2850, 1968.

Romberg, T. A., & Steitz, J. Items to test level of attainment of mathematics concepts by intermediate-grade children. Wisconsin Research and Development Center for Cognitive Learning, Working Paper No. 56, 1971.

Romberg, T. A., Steitz, J., & Frayer, D. A. Selection and analysis of mathematics concepts for inclusion in tests of concept attainment. Wisconsin Research and Development Center for Cognitive Learning, Working Paper No. 55, 1971.

Tucker, L. R. Experiments in multimode factor analysis. In A. Anastasi (Ed.), Testing problems in perspective. Washington, D. C.: American Council on Education, 1966a.

Tucker, L. R. Some mathematical notes on three-mode factor analysis. Psychometrika, 1966b, 31, 279-311.

GPO 82- 030-4

Appendix A
Key for Concepts in Numerical Order

1 Disjoint Sets
2 Empty Set
3 Equal Sets
4 Equivalent Sets
5 Line
6 Parallel Lines
7 Plane
8 Point
9 Subset
10 Subtraction
11 Denominator
12 Division
13 Factor
14 Fraction
15 Mixed Fraction
16 Multiplication
17 Numerator
18 Product
19 Quotient
20 Remainder
21 Average
22 Graph
23 Measurement
24 Open Sentence
25 Place Holder
26 Place Value
27 Solution Set
28 Standard Unit
29 Statement
30 Weight

Appendix B
Orthogonal Common Factor Results for Social Studies Concepts: Boys[a]

Concept	Alpha		Harris R-S[2]								UMLFA				
	A-1	A-2	H-1	H-2	H-3	H-4	H-5	H-6	H-7	H-8	U-1	U-2	U-3	U-4	U-5
Area: Set Theory															
1 Disjoint Sets		56					54					47			
2 Empty Set	44	67		44	47							55		44	
3 Equal Sets	52	60	32		36			49			35	63	39		
4 Equivalent Sets	31	66			35		41					56		32	
5 Line	51	54		56								38		60	
6 Parallel Lines	58	45	32	34				49			36	41	36	34	
7 Plane	44	58				36	34					45	36	34	
8 Point	52	56		39				38				47	31	41	
9 Subset		66			66							61			
10 Subtraction	70	49	47	41		33					45	37	38	43	
Area: Division															
11 Denominator	74		69								72		31		
12 Division	66	47	47	38							47	35		41	33
13 Factor	55	38				60							62		
14 Fraction	70	38	67								68	35			
15 Mixed Fraction	62	55	49		31			34			52	53			
16 Multiplication	64	42	51	34					35		49	31		37	33
17 Numerator	73	31	67								63				
18 Product	72	38	43			55					40		65		
19 Quotient	65	41	43			33			44		42	34	41		33
20 Remainder	61	48	42			32					41	36	34	33	32
Area: Expressing Relationships															
21 Average	49	39						48			32	43	42		
22 Graph	54	43	39	39							39			43	
23 Measurement	63	48	43								45	41	31		
24 Open Sentence	48	52	33				44				35	44			
25 Place Holder	59	54	35	32					44		34	38		36	52
26 Place Value	42	64			43					38		58			33
27 Solution Set	45	64			38					31		58			39
28 Standard Unit	60	46	42				33				41	35	31	37	
29 Statement	61	49	37	31					42		39	38		37	34
30 Weight	59	53	35	49					31		37	40		50	

[a] Includes those variables which have coefficients greater than .30 (absolute).

Decimals have been omitted.

35/36

Appendix C
Orthogonal Common Factor Results for Mathematics Concepts: Girls[a]

Concept	Alpha		Harris R-S[2]							UMLFA		
	A-1	A-2	H-1	H-2	H-3	H-4	H-5	H-6	H-7	U-1	U-2	U-3
Area: Set Theory												
1 Disjoint Sets	54	40	41							38	34	44
2 Empty Set	55	58	40	36						42	46	50
3 Equal Sets	54	53		38		34				50	45	34
4 Equivalent Sets	49	49	57								33	72
5 Line	38	58		32					48	37	47	33
6 Parallel Lines	47	39				58				46		32
7 Plane	37	49					51			41	40	
8 Point	46	61		37			35			44	50	36
9 Subset	64		41		36					51		38
10 Subtraction	65	45			34	44				57	34	43
Area: Division												
11 Denominator		71		64							72	
12 Division	56	49				32				50	41	37
13 Factor	59				63					65		
14 Fraction	41	67	34	58							62	45
15 Mixed Fraction	42	70	32	49						36	61	42
16 Multiplication	62	35			48					63		
17 Numerator		65		68							67	
18 Product	60	44			46					58	37	32
19 Quotient	58	40			44					53	31	36
20 Remainder	59	42	41							45	31	49
Area: Expressing Relationships												
21 Average	55									44		31
22 Graph	45	38					45			39	31	31
23 Measurement	53	43	53							35		57
24 Open Sentence	46	45						51		47	41	
25 Place Holder	54	44	51					34		44		46
26 Place Value	60	38	57		31					45		50
27 Solution Set	55	51	38	35						50	42	38
28 Standard Unit	58	41	37				34			47	32	42
29 Statement	51	55		33		39				49	45	36
30 Weight	36	65	31	38			31		32	32	50	42

[a] Includes those variables which have coefficients greater than .30 (absolute).

Decimals have been omitted.

Appendix D
Orthogonal Common Factor Results for Mathematics Tasks: Boys[a]

Task	Alpha	Harris R-S[2]			UMLFA
	A-1	H-1	H-2	H-3	U-1
1 Given name of attribute, select example.	90	73	43	34	91
2 Given example of attribute, select name.	91	75	47		91
3 Given name of concept, select example.	89	75	45		90
4 Given name of concept, select nonexample.	84	72	44		84
5 Given example of concept, select name.	91	74	49		91
6 Given concept, select relevant attribute.	91	63	58		91
7 Given concept, select irrelevant attribute.	72	36	69		72
8 Given definition of concept, select name.	92	64	56	36	92
9 Given name of concept, select definition.	89	62	53	41	90
10 Given concept, select supraordinate concept.	87	67	53		87
11 Given concept, select subordinate concept.	86	61	59		85
12 Given two concepts, select relationship.	78	48	59		77

[a] Includes those variables which have coefficients greater than .30 (absolute).

Decimals have been omitted.

Appendix E
Orthogonal Common Factor Results for Mathematics Tasks: Girls[a]

Task	Alpha	Harris R-S[2]		UMLFA	
	A-1	H-1	H-2	U-1	U-2
1 Given name of attribute, select example.	87	80	40	84	36
2 Given example of attribute, select name.	89	77	44	79	45
3 Given name of concept, select example.	88	72	52	73	50
4 Given name of concept, select nonexample.	82	73	40	71	44
5 Given example of concept, select name.	88	70	50	69	54
6 Given concept, select relevant attribute.	89	67	58	66	60
7 Given concept, select irrelevant attribute.	63	31	63	31	63
8 Given definition of concept, select name.	91	72	54	70	59
9 Given name of concept, select definition.	88	73	49	72	5
10 Given concept, select supraordinate concept.	85	62	54	63	57
11 Given concept, select subordinate concept.	83	64	48	65	51
12 Given two concepts, select relationship.	75	42	66	41	69

[a] Includes those variables which have coefficients greater than .30 (absolute).

Decimals have been omitted.

GPO 827-030-3

National Evaluation Committee

Helen Bain
Immediate Past President
National Education Association

Lyle E. Bourne, Jr.
Institute for the Study of Intellectual Behavior
Universiy of Colorado

Jeanne S. Chall
Graduate School of Education
Harvard University

Francis S. Chase
Department of Education
University of Chicago

George E. Dickson
College of Education
University of Toledo

Hugh J. Scott
Superintendent of Public Schools
District of Columbia

H. Craig Sipe
Department of Instruction
State University of New York

G. Wesley Sowards
Dean of Education
Florida International University

Benton J. Underwood
Department of Psychology
Northwestern University

Robert J. Wisner
Mathematics Department
New Mexico State University

Executive Committee

William R. Bush
Director of Program Planning and Management
and Deputy Director, R & D Center

Herbert J. Klausmeier, Committee Chairman
Director, R & D Center

Wayne Otto
Principal Investigator
R & D Center

Robert G. Petzold
Professor of Music
University of Wisconsin

Richard A. Rossmiller
Professor of Educational Administration
University of Wisconsin

James E. Walter
Coordinator of Program Planning
R & D Center

Russell S. Way, ex officio
Program Administrator, Title III ESEA
Wisconsin Department of Public Instruction

Faculty of Principal Investigators

Vernon L. Allen
Professor of Psychology

Frank H. Farley
Associate Professor
Educational Psychology

Marvin J. Fruth
Associate Professor
Educational Administration

John G. Harvey
Associate Professor
Mathematics

Frank H. Hooper
Associate Professor
Child Development

Herbert J. Klausmeier
Center Director
V. A. C. Henmon Professor
Educational Psychology

Stephen J. Knezevich
Professor
Educational Administration

Joel R. Levin
Associate Professor
Educational Psychology

L. Joseph Lins
Professor
Institutional Studies

Wayne Otto
Professor
Curriculum and Instruction

Thomas A. Romberg
Associate Professor
Curriculum and Instruction

Peter A. Schreiber
Assistant Professor
English

Richard L. Venezky
Associate Professor
Computer Science

Alan M. Voelker
Assistant Professor
Curriculum and Instruction

Larry M. Wilder
Assistant Professor
Communication Arts